

Find the 5th term in the expansion of $(7n^2 - 3n^5)^{27}$.

SCORE: ____ / 5 PTS

NOTES: Your coefficient may use the operations +, -, × and positive exponents only.
All exponents must be simplified and all signs must be explicit (at the front).

$${}_{27}C_4 (7n^2)^{27-4} (-3n^5)^4 = {}_{27}C_4 (7n^2)^{23} (-3n^5)^4$$

$$= \frac{27!}{4!23!} 7^{23} n^{46} (-3)^4 n^{20}$$

① POINT EACH

★ SUBTRACT ① POINT IF YOUR ANSWER IS NEGATIVE

$$= \frac{27 \cdot 26 \cdot 25 \cdot 24 \cdot \cancel{23!}}{4 \cdot 3 \cdot 2 \cdot 1 \cdot \cancel{23!}} 7^{23} 3^4 n^{66}$$

$$= \boxed{27 \cdot 26 \cdot 25} \cdot \boxed{7^{23}} \cdot \boxed{3^4} \cdot \boxed{n^{66}}$$

Use the entries of Pascal's Triangle to expand and simplify $(9t - 10z)^5$.

SCORE: ____ / 5 PTS

NOTES: Your coefficients may use the operations +, -, × and positive exponents only.
All exponents must be simplified and all signs must be explicit (at the front).
You must show the intermediate step in the expansion to get full credit.

$$1(9t)^5(-10z)^0 + 5(9t)^4(-10z)^1 + 10(9t)^3(-10z)^2 + 10(9t)^2(-10z)^3$$

$$+ 5(9t)^1(-10z)^4 + 1(9t)^0(-10z)^5$$

$$= \boxed{9^5 t^5} - \boxed{5 \cdot 9^4 \cdot 10 t^4 z} + \boxed{10 \cdot 9^3 \cdot 10^2 t^3 z^2} - \boxed{10 \cdot 9^2 \cdot 10^3 t^2 z^3}$$

$$+ \boxed{5 \cdot 9 \cdot 10^4 t z^4} - \boxed{10^5 z^5}$$

① ② POINT EACH ★ SUBTRACT ① POINT IF YOUR NEGATIVES IN THE FINAL ANSWER ARE INSIDE ()'S

Find the rational number representation of the repeating decimal $0.\overline{654}$.

SCORE: ____ / 5 PTS

NOTES: Only the 54 is repeated.
You must use only techniques from sections 9.2-9.5.

$$\boxed{0.6 + 0.054 + 0.00054 + 0.0000054 + \dots} \text{ ①}$$

$$= \frac{6}{10} + \frac{\frac{54}{1000}}{1 - \frac{1}{100}} \text{ ① } \frac{1}{2}$$

$$= \frac{3}{5} + \frac{\frac{54}{1000}}{\frac{99}{100}}$$

$$= \frac{3}{5} + \frac{\frac{3}{55}}{\frac{1}{55}} \text{ ① } \frac{1}{2}$$

$$= \frac{33 + 3}{55} = \frac{36}{55} \text{ ①}$$

AJ and BJ both have sequences where the third term is 81, and the sixth term is -24.

SCORE: ____ / 5 PTS

[a] If AJ's sequence is geometric, find the first term of AJ's sequence.

$$\begin{aligned} a_3 &= a, r^2 = 81 \\ a_6 &= a, r^5 = -24 \end{aligned} \xrightarrow{\textcircled{\frac{1}{2}}} \frac{a, r^5}{a, r^2} = \frac{-24}{81} \xrightarrow{\textcircled{\frac{1}{2}}} r^3 = -\frac{8}{27} \xrightarrow{\textcircled{\frac{1}{2}}} r = -\frac{2}{3}$$

$$a, \left(-\frac{2}{3}\right)^2 = 81 \rightarrow \frac{4}{9}a = 81 \xrightarrow{\textcircled{\frac{1}{2}}} a = \frac{729}{4}$$

[b] If BJ's sequence is arithmetic, find the sum of the first 11 terms of BJ's sequence.

$$\begin{aligned} a_3 &= a + 2d = 81 \\ a_6 &= a + 5d = -24 \end{aligned} \xrightarrow{\textcircled{\frac{1}{2}}} \begin{aligned} a + 2(-35) &= 81 \\ a_1 &= 151 \end{aligned}$$

$$\begin{aligned} -3d &= 105 \\ d &= -35 \end{aligned} \xrightarrow{\textcircled{\frac{1}{2}}} S_{11} = \frac{11}{2}(2(151) + (11-1)(-35)) \xrightarrow{\textcircled{1}}$$

$$= \frac{11}{2}(302 - 350) = \frac{11}{2} \cdot -48 = -264 \xrightarrow{\textcircled{\frac{1}{2}}}$$

Using mathematical induction, prove that $\sum_{i=1}^n [(3i+1) \cdot 4^i] = n \cdot 4^{n+1}$ for all positive integers n .

SCORE: ____ / 10 PTS

BASIS STEP: PROVE $\sum_{i=1}^1 (3i+1) \cdot 4^i = 1 \cdot 4^{1+1} = 1 \cdot 4^2$

$$\sum_{i=1}^1 (3i+1) \cdot 4^i = (3(1)+1) \cdot 4^1 = 4 \cdot 4 = 16$$

$$1 \cdot 4^2 = 16 \checkmark$$

INDUCTIVE STEP: ASSUME $\sum_{i=1}^k (3i+1) \cdot 4^i = k \cdot 4^{k+1}$ FOR SOME PARTICULAR BUT ARBITRARY

INTEGER $k \geq 1$

PROVE $\sum_{i=1}^{k+1} (3i+1) \cdot 4^i = (k+1) \cdot 4^{(k+1)+1} = (k+1) \cdot 4^{k+2}$

$$\begin{aligned} \sum_{i=1}^{k+1} (3i+1) \cdot 4^i &= \sum_{i=1}^k (3i+1) \cdot 4^i + (3(k+1)+1) \cdot 4^{k+1} \\ &= k \cdot 4^{k+1} + (3k+4) \cdot 4^{k+1} \\ &= (k+3k+4) \cdot 4^{k+1} \\ &= (4k+4) \cdot 4^{k+1} \\ &= 4(k+1) \cdot 4^{k+1} \\ &= (k+1) \cdot 4^{k+2} \end{aligned}$$

GRADED
BY ME

BY MI, $\sum_{i=1}^n (3i+1) \cdot 4^i = n \cdot 4^{n+1}$ FOR ALL POSITIVE INTEGERS n